Name:____ Date:_______________ Period:_____

1.1 – Graphs of Equations

Objectives:

- Sketch graphs of equations
- Find x- and y-intercepts of graphs of equations
- Use symmetry to sketch graphs of equations
- Find equations of and sketch graphs of circles
- Use graphs of equations in solving real-life problems

THE GRAPH OF AN EQUATION

A relationship between two quantities is expressed as an **equation in two variables.** An ordered pair (a, b) is a **solution** or **solution point** of an equation in x and y if the equation is true when a is substituted for x, and b is substituted for y.

In this section, you will review some basic procedures for sketching the graph of an equation in two variables. The **graph of an equation** is the set of all points that are solutions of the equation.

1. Determine whether the point lies on the graph $y = 14 - 6x$.

a. $(3, -5)$

b. $(-2, 26)$

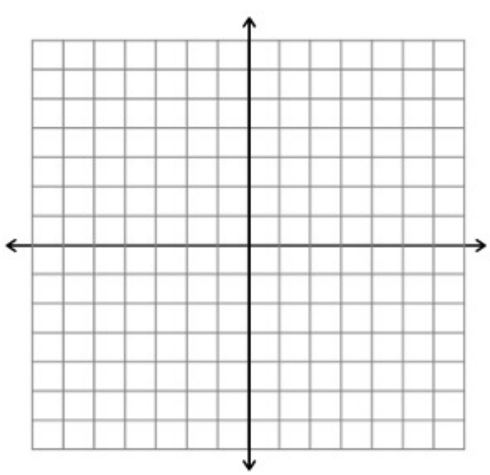
The most basic technique used for sketching the graph of an equation is the **point-plotting method.**

Sketching the Graph of an Equation by Point Plotting
1. If possible, rewrite the equation so that one of the variables is isolated on one side of the equation.
2. Make a table of values showing several solution points. Five points are generally enough.
3. Plot these points on a rectangular coordinate system.
4. Connect the points with a smooth curve or line.

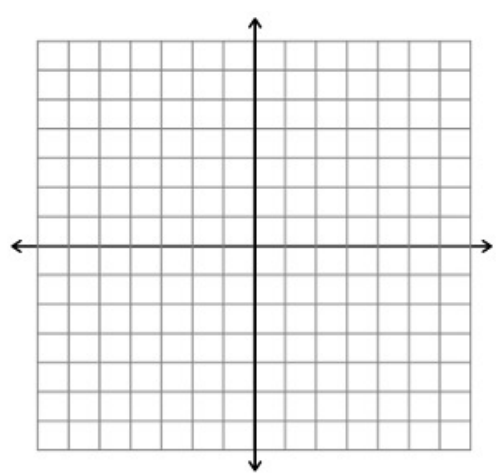
NOTE: When making a table of solution points, be sure to use positive, zero, and negative values of x!

2. Sketch the graph of the following equations.

a. $3x + y = 2$

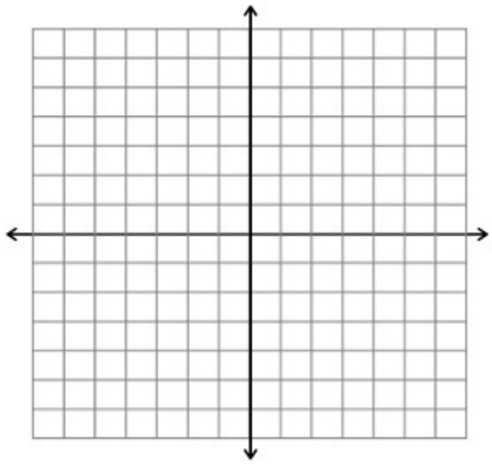


b. $-2x + y = 1$

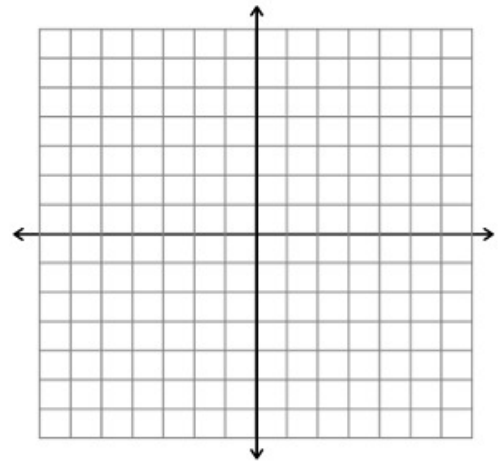


3. Sketch the graph of the following equations.

a. $y = x^2 + 3$



b. $y = 1 - x^2$

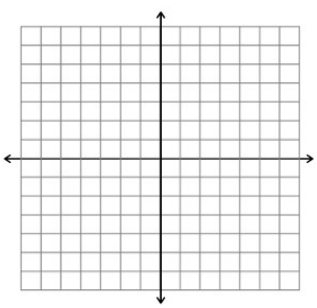


INTERCEPTS OF A GRAPH

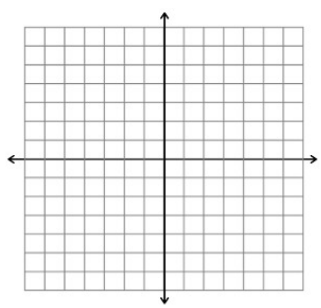
It is often easy to determine the solution points that have zero as either the x-coordinate or the y-coordinate. These points are called **intercepts** because they are the points at which the graph intersects or touches the x- or y-axis. It is possible for a graph to have no intercepts, one intercept, or several intercepts.

4. Sketch a graph with the given

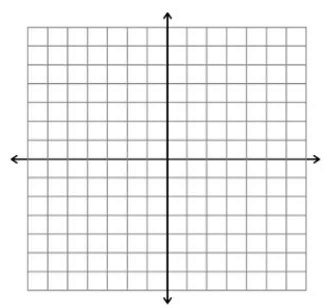
No x-intercepts
One y-intercept



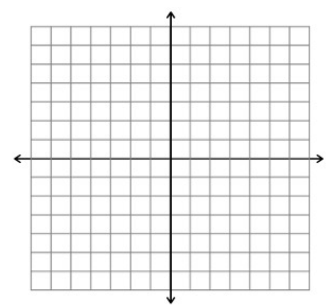
Three x-intercepts
One y-intercept



One x-intercept
Two y-intercepts



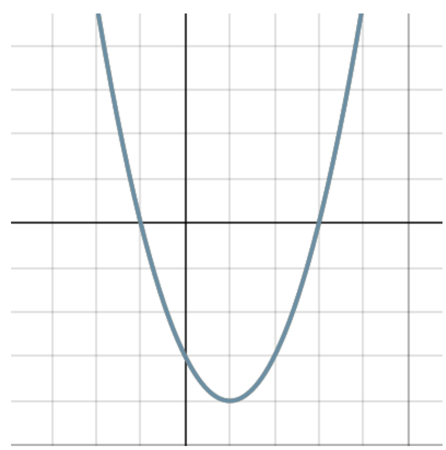
No x-intercepts
No y-intercepts



5. Identify the x- and y-intercepts of the graph

a.

b.

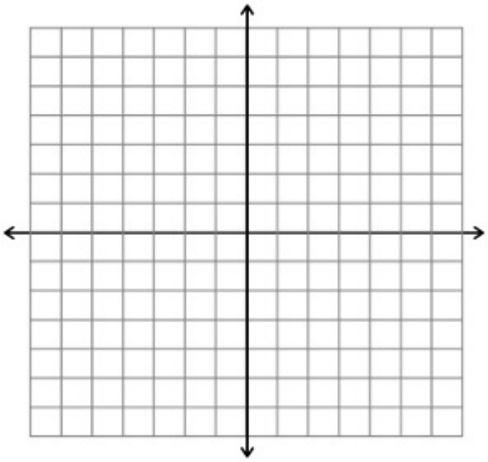


SYMMETRY

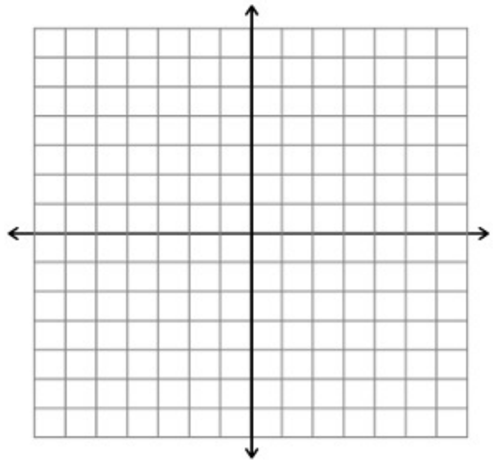
Graphs of equations can have **symmetry** with respect to one of the coordinate axes or with respect to the origin. Symmetry with respect to the x-axis, y-axis, or origin means that if the plane were folded along the x-axis, y-axis, or origin.

6. Sketch a graph that correlates with the type of symmetry.

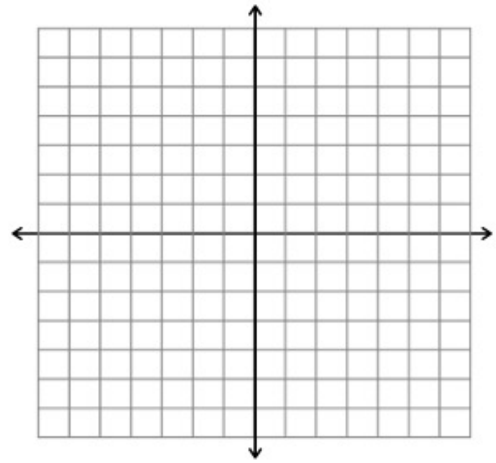
x-axis Symmetry



y-axis Symmetry



Origin Symmetry



NOTE: Knowing the symmetry of a graph before attempting to sketch a graph is helpful because then you only need half as many points.

Graphical Tests for Symmetry

1. A graph is **symmetric with respect to the x-axis** if, whenever (x, y) is on the graph, $(x, -y)$ is also on the graph.
2. A graph is **symmetric with respect to the y-axis** if, whenever (x, y) is on the graph, $(-x, y)$ is also on the graph.
3. A graph is **symmetric with respect to the origin** if, whenever (x, y) is on the graph, $(-x, -y)$ is also on the graph.

7. Determine the symmetry of each table of points.

a.

x	-2	-1	0	1	2
y	8	5	4	5	8

b.

x	-2	-1	0	1	2
y	-14	-12	-10	-8	-6

c.

x	-2	-1	0	1	2
y	10	2	0	-2	-10

d.

x	0	-3	4	3	0
y	-2	-1	0	1	2

Algebraic Tests for Symmetry

1. A graph of an equation is symmetric with respect to the x-axis if replacing y with $-y$ yields an equivalent equation.
2. A graph of an equation is symmetric with respect to the y-axis if replacing x with $-x$ yields an equivalent equation.
3. A graph of an equation is symmetric with respect to the origin if replacing x with $-x$ and y with $-y$ yields an equivalent equation.

8. Test to see if the equation is symmetrical to the x-axis, y-axis, or origin.

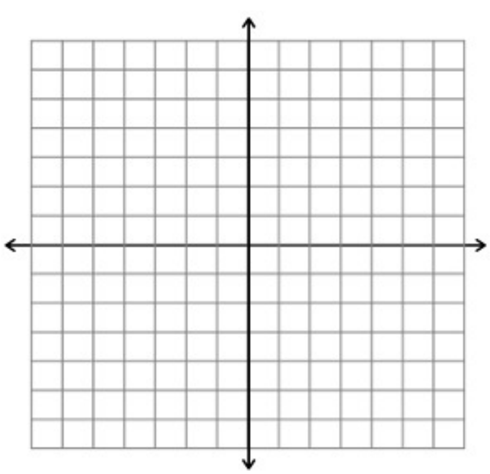
a. $y^2 = 6 - x$

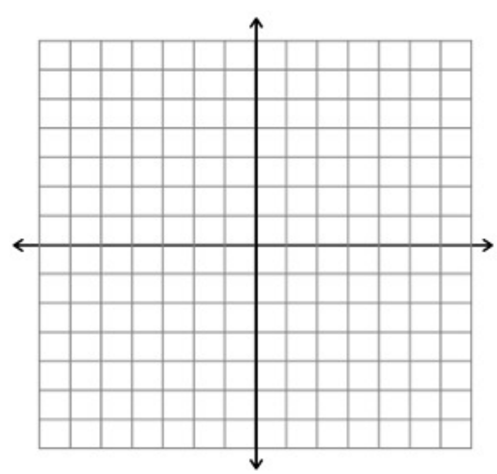
b. $y = x^2 + 4$

9. Sketch a graph of each equation using symmetry.

a. $y = -x^2 - 2x$

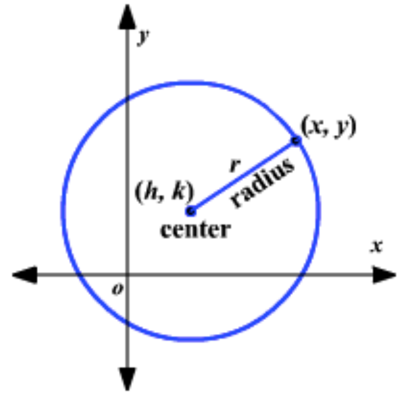
b. $y = 1 - |x|$





CIRCLES

A point (x, y) is on the circle if and only if its distance from the center (h, k) is r.



By the distance formula, $\sqrt{(x-h)^2 + (y-k)^2} = r$.

By squaring each side of this equation, you obtain the **standard form of the equation of a circle.**

Standard Form of the Equation of a Circle

The point (x, y) lies on the circle of **radius** r and **center** (h, k) if and only if:

$$(x-h)^2 + (y-k)^2 = r^2$$

---

10. Find the equation of each circle given a point and the center.

a. Point: $(1, -2)$
Center: $(-3, -5)$

b. Point: $(-1, 1)$
Center: $(3, -2)$

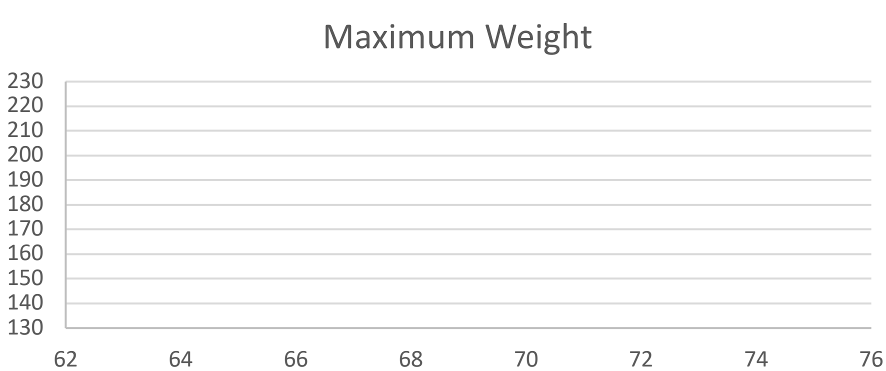
APPLICATION

11. The maximum weight y (in pounds) for a man in the United States Marine Corps can be approximated by the mathematical model, $y = 0.040x^2 - 0.11x + 3.9,\ \ 58 \leq x \leq 80$ where x is the man's height (in inches).

a. Construct a table of values that shows the maximum weights for men with heights of 62, 64, 66, 68, 70, 72, 74, and 76 inches.

Height, x	62	64	66	68	70	72	74	76
Weight, y								

b. Use the table of values from part a to sketch a graph of the model. Then use the graph to estimate graphically the maximum weight for a man whose height is 75 inches.



c. Use the model to confirm algebraically the estimate you found from part b.

1.2 – Linear Equations in One Variable

Objectives:

- Identify different types of equations
- Solve linear equations in one variable
- Solve equations that lead to linear equations
- Find x- and y-intercepts of graphs of equations algebraically
- Use linear equations to model and solve real-life problems

EQUATIONS AND SOLUTIONS OF EQUATIONS

An **equation** in x is a statement that two algebraic expressions are equal. To solve an equation in x means to find all values of x for which the equation is true. These values are called **solutions.**

The solutions of an equation depend on the type of numbers being considered. For example, $x^2 = 10$ has no rational solution because there are no rational numbers whose square is 10. However, the same equation has two real solutions, $x = \sqrt{10}$ and $x = -\sqrt{10}$.

An equation that is true for every real number in the domain of the variable is called an **identity**. An equation is an identity when both sides are equivalent. An equation that is true for just some (or none) of the real numbers in the domain of the variable is called a **conditional equation**.

LINEAR EQUATIONS IN ONE VARIABLE

Definition of Linear Equation

A **linear equation in one variable** x is an equation that can be written in the standard form:

$$ax + b = 0$$

where a and b are real numbers with $a \neq 0$.

To solve a conditional equation in x, isolate x on one side of the equation by a sequence of **equivalent equations,** each having the same solution(s) as the original equation.

Generating Equivalent Equations

An equation can be transformed into an equivalent equation by one or more of the following steps:

	GIVEN EQUATION	EQUIVALENT EQUATION
1. Remove symbols of grouping, combine like terms, or simplify fractions on one or both sides of the equation.	$2x - x = 4$	$x = 4$
2. Add (or subtract) the same quantity to (from) each side of the equation.	$x + 1 = 6$	$x = 5$
3. Multiply (or divide) each side of the equation by the same nonzero quantity.	$2x = 6$	$x = 3$
4. Interchange the two sides of the equation.	$2 = x$	$x = 2$

1. Solve each linear equation.

a. $7 - 2x = 15$

b. $7x - 9 = 5x + 7$

c. $4(x + 2) - 12 = 5(x - 6)$

d. $4(-8x + 5) = -32x - 26$

EQUATIONS THAT LEAD TO LINEAR EQUATIONS

To solve an equation involving fractional expressions, find the least common denominator (LCD) of all terms and multiply every term by the LCD. This process will clear the original equation of fractions and produce a simpler equation to work with.

NOTE: You can also find a common denominator and combine the fractions if you wish to do so.

2. Solve each equation by getting rid of the fractions.

a. $\frac{4x}{9} - \frac{1}{3} = x + \frac{5}{3}$

b. $\frac{3x}{4} - \frac{7}{2} = \frac{5}{6}$

c. $\frac{2x}{3} - 2 = \frac{3x}{2} + \frac{1}{6}$

d. $\frac{x-5}{6} = \frac{x}{4} - 1$

When there is a variable in the denominator, it is possible to have an **extraneous solution**. An extraneous solution is one that does not satisfy the original equation.

3. Find the extraneous solution and solve for each variable.

a. $\frac{1}{6x^2} = \frac{1}{3x^2} - \frac{1}{x}$

b. $\frac{2}{x-3} + \frac{1}{x} = \frac{x-1}{x-3}$

c. $\frac{3x}{x-4} = 5 + \frac{12}{x-4}$

d. $\frac{p+5}{p^2+p} = \frac{1}{p^2+p} - \frac{p-6}{p+1}$

FINDING INTERCEPTS ALGEBRAICALLY

To find x- and y-intercepts algebraically, it depends on which intercept you are finding to determine how to solve.

Finding Intercepts Algebraically
1. To find x-intercepts, set y equal to zero and solve the equation for x.
2. To find y-intercepts, set x equal to zero and solve the equation for y.

4. Find the x- and y-intercepts of each equation.

a. $y = -3x - 2$

b. $5x + 3y = 15$

APPLICATION

5. The number y (in thousands) of female participants in high school athletic programs in the United States from 2008 through 2015 can be approximated by the linear model, $y = 3.66t + 91.4, -2 \leq t \leq 5,$ where $t = 0$ represents 2010.

a. Find algebraically the y-intercept of the linear model.

b. Assuming this linear pattern continues, find the year in which there will be 128,000 female participants.

1.3 – Modeling with Linear Equations

Objectives:

- Write and use mathematical models to solve real-life problems
- Solve mixture problems
- Use common formulas to solve real-life problems

INTRODUCTION TO PROBLEM SOLVING

The process of translating phrases or sentences into algebraic expressions or equations is called **mathematical modeling.** To use mathematical modeling, start by using the verbal description of the problem to form a verbal model. Then, after assigning labels to the quantities in the verbal model, form a mathematical model or algebraic equation.

NOTE: When you are constructing a verbal model, it is helpful to look for a hidden equality.

1. You have accepted a job for which your annual salary will be $58,400. This includes your salary and a year-end bonus of $1,200. You will be paid twice a weekly. What is your salary per pay period?

2. You buy stock at $15 per share. You sell the stock at $18 per share. What is the percent increase in the stock's value?

3. Your family has annual loan payments equal to 28% of its annual income. During the year, the loan payments total is $17,920. What is your family's annual income?

4. A rectangular room is three times as long as it is wide, and its perimeter is 112 feet. Find the dimensions of the room.

5. A plane is flying nonstop from Portland, Oregon, to Atlanta, Georgia, a distance of about 2170 miles. After 3 hours in the air, the plane flies over Topeka, Kansas (a distance of about 1440 miles from Portland). Assuming the plane flies at a constant speed, how long does the entire trip take?

6. You measure the shadow cast by a building and find that it is 55 feet long. Then you measure the shadow case by a nearby four-foot post and find that it is 1.8 feet long. Determine the building's height.

MIXTURE PROBLEMS

Problems that involve two or more rates are called mixture problems.

7. You invested a total of $5,000 at 2.5% and 3.5% simple interest. During one year, the two accounts earned $151.25. How much did you invest in each account?

8. A store has $30,000 of inventory in single-disc DVD players and multi-disc DVD players. The profit on a single-disk player is 22% and the profit on a multi-disc player is 40%. The profit for the entire stock is 35%. How much was invested in each type of DVD player?

COMMON FORMULAS

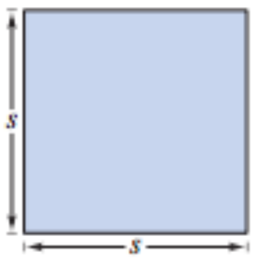
A **literal equation** is an equation that contains more than one variable. A formula is an example of a literal equation. Many common types of geometric, scientific, and investment problems use ready-made formulas. Knowing these formulas will help you translate and solve a variety of real-life equations.

Common Formulas for Area, Perimeter, Circumference, and Volume

SQUARE

$A = s^2$

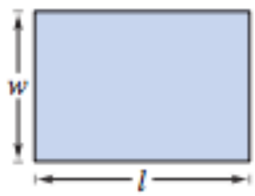
$P = 4s$



RECTANGLE

$A = lw$

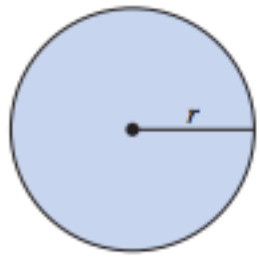
$p = 2l + 2w$



CIRCLE

$A = \pi r^2$

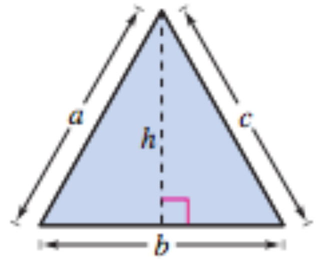
$C = 2\pi r$



TRIANGLE

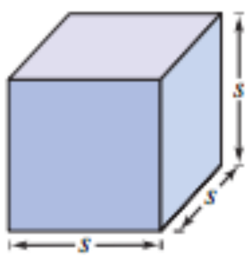
$A = \frac{1}{2}bh$

$P = a + b + c$



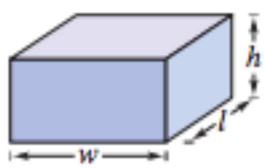
CIRCLE

$V = s^3$



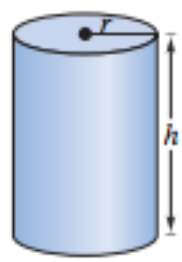
RECTANGULAR SOLID

$V = lwh$



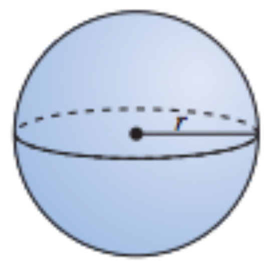
CIRCULAR CYLINDER

$V = \pi r^2 h$



SPHERE

$V = \frac{4}{3}\pi r^3$



9. A cylindrical container has a volume of 84 cubic inches and a radius of 3 inches. Find the height of the container.

10. A billiard ball has a volume of 5.96 cubic inches. Find the radius of a billiard ball.

1.4 – Quadratic Equations and Applications

Objectives:

- Solve quadratic equations by factoring
- Solve quadratic equations by extracting square roots
- Solve quadratic equations by completing the square
- Use the Quadratic Formula to solve quadratic equations
- Use quadratic equations to model and solve real-life problems

FACTORING

In lesson P.4, we learned how to solve quadratic functions by factoring. Let's review!

1. Solve each quadratic function by factoring.

a. $2x^2 - 3x + 1 = 6$

b. $6x^2 - 3x = 0$

EXTRACTING SQUARE ROOTS

2. When you take the square root of a number or a variable, why does it need a plus or minus sign?

3. Solve for the variable by extracting the square root.

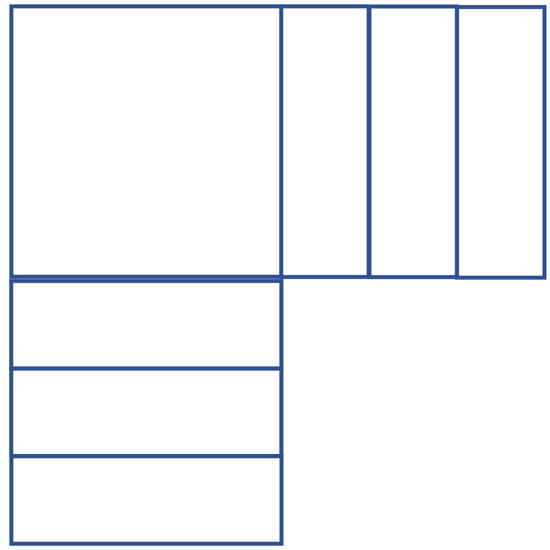
a. $3x^2 = 36$

b. $(x - 1)^2 = 10$

COMPLETING THE SQUARE

If you have a quadratic equation that cannot be factored but you still need to solve for the variable, one way to do that is by **completing the square**.

4. This partial square on the right represents $x^2 + 6x$.
   a. Mark each rectangle/square with the algebraic expression it represents.



   b. How many unit tiles would you need to complete the square?

   c. What are the dimensions of the completed square?

   d. Replace c and the question mark to make the statement true.

   $x^2 + 6x + c = (x+?)^2$ $\qquad$ $c =$ ________ $\qquad$ $? =$ ________

5. Find the missing c in each problem and then rewrite the trinomial as a perfect square binomial.
   a. $x^2 + 12x + c$ $\qquad$ b. $x^2 - 10x + c$

   c. $x^2 + 3x + c$ $\qquad$ b. $x^2 + bx + c$

Using the steps below, solve $x^2 + 10x - 39 = 0$.

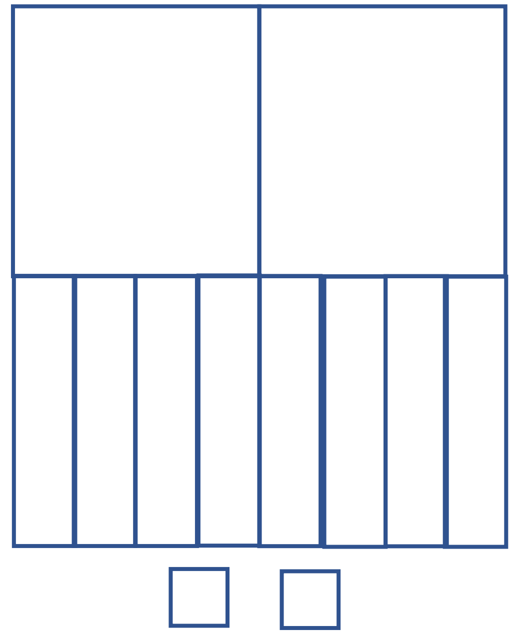
STEPS	ALGEBRA WORK
1. Move the c value to the right-hand side of the equal sign.	
2. Find the value that makes the left-hand side of the equal sign a perfect square trinomial.	
3. Add that value to both sides of the equation.	
4. Rewrite the equation at a perfect square binomial.	
5. Take the square root of both sides. NOTE: Don't forget the $\pm$ sign!	
6. Finish solving for the variable.	

6. Solve each equation by completing the square.

a. $x^2 - 4x - 1 = 0$

b. $x^2 + 6x = 12$

The diagram to the right is representing what expression?



What would it look like if we split the diagram up into two equal groups?

How many tiles are we missing to make two complete squares?

We can make that into the equation $2(x+2)^2 - 6$. Why would it be a negative 6?

7. Solve $2(x+2)^2 - 6 = 0$.

Using the steps below, solve $8x^2 + 16x = 42$.

STEPS	ALGEBRA WORK
1. Move the c value to the right-hand side of the equal sign.	
2. Factor out the common number of the variable terms.	
3. Find the value that makes the set of parentheses on the left-hand side of the equal sign a perfect square trinomial.	
4. Think about what number would need to be added to both sides in order to make it equal on both sides. Keep in mind the number you factored out!	
5. Factor the left-hand side and simplify the right-hand side.	
6. Divide by the factored-out term.	
7. Take the square root of both sides. NOTE: Don't forget the $\pm$ sign!	
8. Finish solving for the variable.	

8. Solve by completing the square.

a. $2x^2 - 4x + 1 = 0$

b. $3x^2 - 10x - 2 = 0$

c. $2x^2 - 8x = -6$

d. $6x^2 + 12x = 48$

e. $3x^2 - 8x + 4 = 0$

f. $x^2 - 10x + 26 = 8$

THE QUADRATIC FORMULA

We have learned two ways to solve for a variable of a quadratic equation, factoring and completing the square. Factoring is the fastest method, but only works part of the time. Completing the square works for every quadratic equation but is not the easiest method. We will now learn a third method that works for every quadratic equation and it relatively simple.

The Quadratic Equation

The solutions of a quadratic equation in the general form

$$ax^2 + bx + c = 0,\ a \neq 0$$

are given by the **quadratic formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

9. Solve each quadratic equation using the quadratic formula.

a. $2x^2 + 2x - 12 = 0$

b. $x^2 - 4x + 4 = 0$

c. $3x^2 + 2x = 10$

d. $4x^2 + 8x - 1 = 0$

e. $9x^2 - 7x - 4 = 0$

f. $5x^2 + 3x + 1 = 0$

In the quadratic formula, the quantity under the radical sign, $b^2 - 4ac$, is called the **discriminant** of the quadratic expression $ax^2 + bx + c = 0$. It can be used to determine the nature of the solutions of a quadratic equation.

<u>Solutions of a Quadratic Equation</u>

The solutions of a quadratic equation $ax^2 + bx + c = 0,\ a \neq 0$, can be classified as follows. If the discriminant $b^2 - 4ac$ is:

1. POSITIVE, then the quadratic equation has two distinct real solutions and its graph has two x-intercepts.
2. ZERO, then the quadratic equation has one repeated real solution and its graph has one x-intercept.
3. NEGATIVE, then the quadratic equation has no real solutions and its graph has no x-intercepts.

10. Find the discriminant of each quadratic equation then state the number of solutions.

a. $9x^2 - 3x - 8 = 0$

b. $-x^2 - 6x - 9 = 0$

c. $2x^2 - 10x - 5 = 0$

d. $x^2 + 3x - 9 = 0$

<u>APPLICATIONS</u>

11. A bedroom is 6 feet longer than it is wide and has an area of 112 square feet. Find the dimensions of the room.

12. From 2000 through 2008, the estimated number of internet users I (in millions) in the United States can be modeled by the quadratic equation, $I = -1.446t^2 + 23.45t + 122.9,\ 0 \leq t \leq 8$, where t represents the year, with $t = 0$ corresponding to 2000. In which year did the number of internet users reach or surpass 200 million?

1.5 – Complex Numbers

Objectives:
- Use the imaginary unit i to write complex numbers
- Add, subtract, and multiply complex numbers
- Use complex numbers to write the quotient of two complex numbers in standard form
- Find complex solutions of quadratic equations

THE IMAGINARY UNIT i

In the last lesson, we learned that some quadratic equations have no real solutions. This happens when you have a negative value underneath the square root. To overcome this issue, mathematicians created an expanded system of numbers using the **imaginary unit i.**

$$i = \sqrt{-1}$$

i	
i^2	
i^3	
i^4	

i^5	
i^6	
i^7	
i^8	

i^{23}	
i^{50}	
i^{101}	
i^{220}	

There are two methods you can follow to find the value of each imaginary expressions.

Method 1:

Method 2:

By adding real numbers to real multiples of this imaginary unit, the set of **complex numbers** is obtained. Each complex number can be written in **standard form** $\boldsymbol{a+bi}$**.**

In the standard form $a+bi$, the real number a is called the real part, and the number bi (where b is a real number) is called the imaginary number.

1.

	COMPLEX NUMBER	REAL PART	IMAGINARY PART
A.	$-2+7i$		
B.	$3-4i$		
C.		-2	$5i$
D.		-7	0
E.		0	$-i$

2. Simplify each expression into complex form.

a. $\sqrt{-24}$

b. $\frac{5\pm\sqrt{-75}}{5}$

c. $\frac{2\pm\sqrt{-20}}{2}$

d. $\frac{1\pm\sqrt{4-16}}{4}$

OPERATIONS WITH COMPLEX NUMBERS

To add or subtract two complex numbers, you add or subtract the real and imaginary parts of the numbers separately.

Addition and Subtraction of Complex Numbers

If $a + bi$ and $c + di$ are two complex numbers written in standard from, their sum and difference are defined as follows:

SUM: $(a + bi) + (c + di) = (a + c) + (b + d)i$

DIFFERENCE: $(a + bi) - (c + di) = (a - c) + (b - d)i$

The **additive identity** in the complex number system is zero (the same as in the real number system). Furthermore, the **additive inverse** of the complex number $a + bi$ is $-(a + bi) = -a - bi$**.** So, you have $(a + bi) + (-a - bi) = 0 + 0i = 0$**.**

3. Simplify the complex numbers by adding or subtracting.

a. $(7 + 3i) + (5 - 4i)$

b. $(3 + 4i) - (5 - 3i)$

c. $2i + (-3 - 4i) - (-3 - 3i)$

d. $(5 - 3i) + (3 + 5i) - (8 + 2i)$

4. Simplify the complex numbers by multiplying.

a. $-5(3 - 2i)$

b. $(2 - 4i)(3 + 3i)$

c. $(4 + 5i)(4 - 5i)$

d. $(4 + 2i)^2$

<u>COMPLEX CONJUGATES</u>

In example 4c, the product of the two complex numbers turned out to be a real number. This occurs with pairs of complex numbers of the form $a + bi$ and $a - bi$. These are called **complex conjugates.**

5. Multiply each complex number by its complex conjugate.

a. $3 + 6i$

b. $2 - 5i$

To divide complex numbers, we want to make sure that we do not have a complex number in the denominator. What would you need to multiply the denominator of $\frac{2+i}{2-i}$ by to get the denominator to be a whole number?

6. Divide each of the complex numbers.

a. $\frac{2+i}{2-i}$

b. $\frac{2-6i}{2+5i}$

c. $\frac{5i}{6+8i}$

d. $\frac{3+9i}{-6-6i}$

COMPLEX SOLUTIONS OF QUADRATICS EQUATIONS

7. Write each complex number in standard form.

a. $\sqrt{-14}\sqrt{-2}$

b. $\sqrt{-48}-\sqrt{-27}$

c. $(-1+\sqrt{-3})^2$

d. $(3+\sqrt{-5})(7-\sqrt{-10})$

8. Solve for the variable of each quadratic equation.

a. $x^2+4=0$

b. $8x^2+14x+9=0$

c. $10x^2-4x+10=0$

d. $6x^2-8x+6=0$

1.6 – Other Types of Equations

Objectives:

- Solve polynomial equations of degree three or greater
- Solve equations involving radicals
- Solve equations involving fractions or absolute values
- Use polynomial equations and equations involving radicals to model and solve real-life problems

POLYNOMIALS EQUATIONS

1. Solve each polynomial equation by factoring.

a. $9x^4 - 12x^2 = 0$

b. $x^3 - 5x^2 - 2x + 10 = 0$

Sometimes, mathematical models involve equations that are of **quadratic type**. In general, an equation is of quadratic type if it can be written in the form

$$au^2 + bu + c = 0$$

where $a \neq 0$ and $u$is an algebraic expression.

2. Solve each quadratic type of equation.

a. $x^4 - 7x^2 + 12 = 0$

b. $9x^4 - 37x^2 + 4 = 0$

EQUATIONS INVOLVING RADICALS

Operations such as squaring each side, raising each side to a power, and multiplying each side of an equation by a variable can introduce extraneous solutions. When using any of these operations, checking your solutions is crucial!

Using the steps below, solve $-\sqrt{40-9x}+2=x$.

STEPS	ALGEBRAIC WORK
1. Isolate the radical. Make sure that one radical is alone on one side of the equation.	
2. Raise both sides of the equation to the power that is the same number as the index of the radical.	
3. Solve for the variable. If there is still a radical, repeat steps 1 and 2.	
4. Check your solutions to see if they are extraneous.	

3. Solve each equation involving radicals.

a. $\sqrt[3]{4x-8}-4=0$

b. $3\sqrt{2x-1}-9=-3$

4. Solve each equation involving multiple radials.

a. $\sqrt{2x-5} - \sqrt{x-3} = 1$

b. $\sqrt{x+7} = 2 - \sqrt{x-5}$

When solving equations with rational exponents, there are two methods to solve for the variable.

Solve $(x-5)^{\frac{2}{3}} = 16$ using both methods.

Method 1:	Method 2:
1. Isolate the term with the power.	1. Convert the rational exponent to radical form.
2. Raise both sides of the equation to the reciprocal power.	2. Isolate the radical.
3. Solve for the variable.	3. Raise both sides of the equation to the index of the radical.
4. Check your answer.	4. Solve for the variable.
	5. Check your answer.

5. Solve each equation involving a rational exponent using any method.

a. $3(5x-1)^{\frac{1}{2}} - 2 = 0$

b. $(3x+2)^{\frac{1}{3}} + 1 = 0$

c. $4(3x+5)^{\frac{2}{3}} = 100$

d. $3(x+2)^{\frac{3}{4}} + 6 = 30$

EQUATIONS WITH FRACTIONS

To solve an equation involving fractions, multiply each side of the equation by the least common denominator (LCD) of all terms in the equation. This will "clear the equation of fractions."

6. Solve each equation involving fractions.

a. $\frac{4}{x} + \frac{2}{x+3} = -3$

b. $\frac{4}{x+1} - \frac{3}{x+2} = 1$

<u>EQUATIONS WITH ABSOLUTE VALUES</u>

To solve an equation involving an absolute value, the expression inside the absolute value signs can be positive or negative. This results in TWO separate equations, each of which needs to be solved.

Using the steps below, solve $|x^2 + 6x| = 2x + 18$

STEPS	ALGEBRAIC WORK
1. Isolate the term with the absolute value.	
2. Make two equations. One equation is with a positive expression from the absolute value. The other equation is with a negative expression from the absolute value.	
3. Solve for the variable in each equation.	
4. Check your solutions to see if they are extraneous.	

7. Solve each equation involving an absolute value.

a. $|x^2 + 4x| = 7x + 18$

b. $|x - 15| = x^2 - 15x$

1.7 – Linear Inequalities in One Variable

Objectives:

- Represent solutions of linear inequalities in one variable
- Use properties of inequalities to create equivalent inequalities
- Solve linear inequalities in one variable
- Solve inequalities involving absolute values
- Use inequalities to model and solve real-life problems

INTRODUCTION

You solve an inequality for a variable by finding all the values of the variable that make the inequality true. Those values are solutions that satisfy the inequality. For when there is more than one value that makes the inequality true, that is called a **solution set.** The set of all points on a real number line that represents the solution set is the graph of the inequality. Graphs of many types of inequalities consist of intervals on the real number line.

1. Write an inequality to represent each interval, and state whether the interval is bounded or unbounded.

a. $[-1,3]$

b. $[0,\infty)$

c. $(-1,6)$

d. $(-\infty,4)$

PROPERTIES OF INEQUALITIES

Solving linear inequalities with one variable is much like solving linear equations. To isolate the variable, you use the **properties of inequalities.**

These properties are similar to the properties of equality, but there are two exceptions. When each side of the inequality is multiplied or divided by a negative number, the inequality symbol must switch directions.

Properties of Inequalities

Let a, b, c and d be real numbers.

1. Transitive Property

$a < b$ and $b < c$ ➔ $a < c$

2. Addition of Inequalities

$a < b$ and $c < d$ ➔ $a + c < b + d$

3. Addition of Constant

$a < b$ ➔ $a + c < b + c$

4. Multiplication by a Constant

For $c > 0$, $a < b$ ➔ $ac < bc$

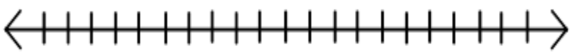
For $c < 0$, $a < b$ ➔ $ac > bc$

SOLVING A LINEAR INEQUALITY IN ONE VARIABLE

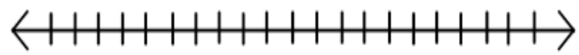
The simplest type of inequality is a linear inequality with one variable.

1. Solve and graph each inequality.

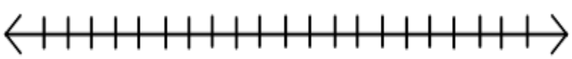
a. $7x - 3 \leq 2x + 7$



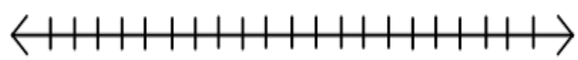
b. $2 - \frac{5}{3}x > x - 6$



c. $2x - 2 \leq 8x + 4$

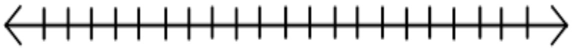


d. $26 + x \geq 5(-6 + 3x)$

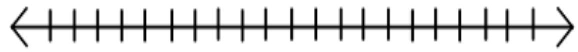


2. Solve and graph each double inequality.

a. $1 \leq 2x + 7 < 11$



b. $-4 \leq 5x - 2 < 7$



INEQUALITIES INVOLVING ABSOLUTE VALUES

Solving an Absolute Value Inequality

Let x be a variable or an algebraic expression and let a be a real number such that $a \geq 0$.

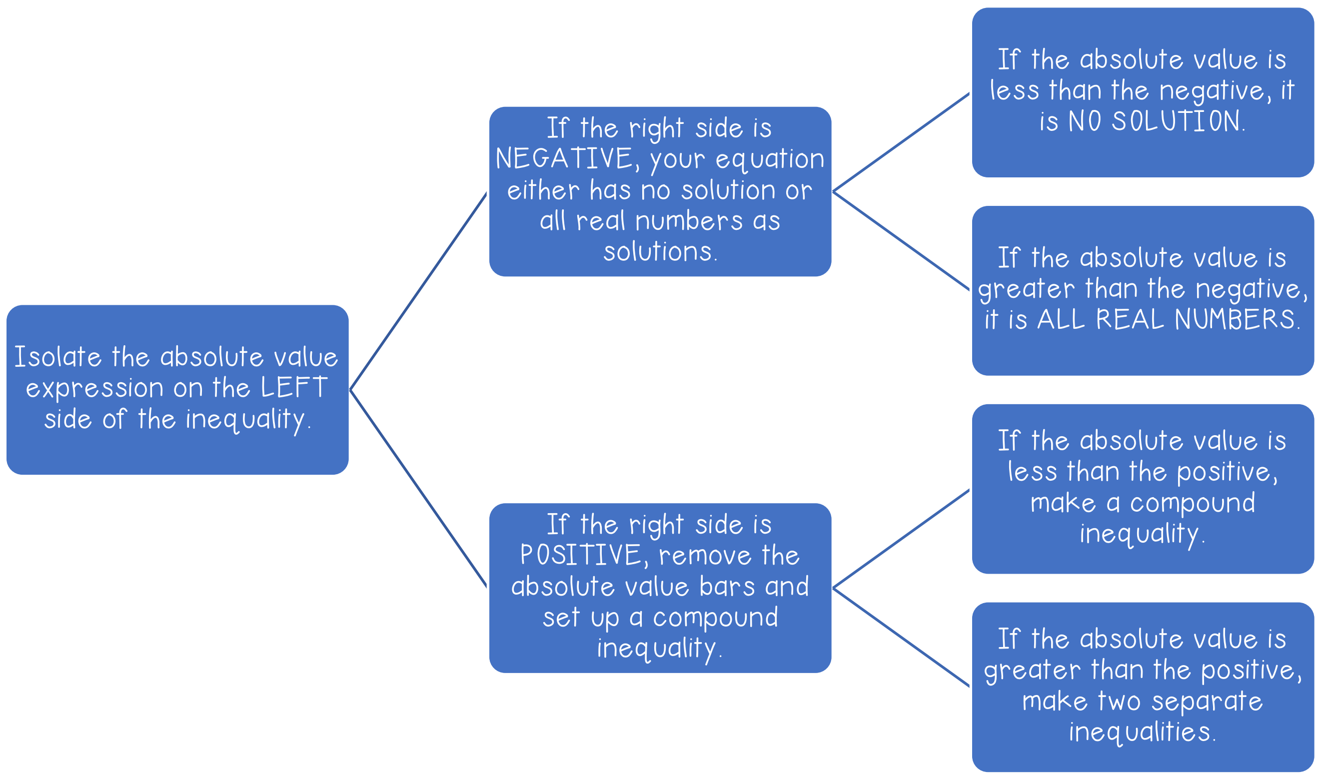
1. The solutions of $|x| < a$ are all values of x that lie between $-a$ and a.

   $|x| < a$ if and only if $-a < x < a$

2. The solutions of $|x| > a$ are all values of x that are less than $-a$ or greater than a.

   $|x| > a$ if and only if $x < -a$ or $x > a$

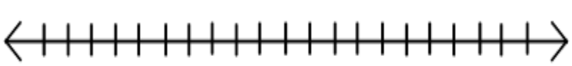
These rules are also valid if $<$ is replaced by $\leq$ and $>$ is replaced by $\geq$.

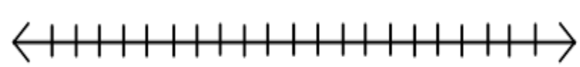


3. Solve and graph each absolute value inequality.

   a. $|x - 20| \leq 4$

   b. $|x + 3| \geq 7$





APPLICATION

4. A car can be rented from Company A for $180 per week with no extra charge for mileage. A similar car can be rented from Company B for $100 per week plus 20 cents for each mile driven. How many miles must you drive in a week in order for the rental fee for Company B to be more than that for Company A?

5. The average salary S (in thousands of dollars) for teachers in the United States from 1990 through 2005 are approximated by the model $S = 1.09t + 30.9, 0 \leq t \leq 15$, where t represents the year, with $t = 0$ corresponding to 1990.

a. According to this model, when was the average salary at least $32,500, but not more than $42,000?

b. According to this model, when will the average salary exceed $54,000?

1.8 – Other Types of Inequalities

Objectives:

- Solve polynomial inequalities
- Solve rational inequalities
- Use inequalities to model and solve real-life problems

POLYNOMIAL INEQUALITIES

To solve a polynomial inequality, you can use the fact that a polynomial can change signs only at its zeros. Between two consecutive zeros, a polynomial must be entirely positive or entirely negative. This means that when the real zeros of a polynomial are put in order, they divide the real number line into intervals in which the polynomial has no sign changes. These zeros are the **key numbers** of the inequality, and the resulting intervals are the **test intervals** for the inequality.

Finding Test Intervals for a Polynomial

To determine the intervals on which the values of a polynomial are entirely negative or entirely positive, use the following steps.

1. Find all real zeros of the polynomial, and arrange the zeros in increasing order (from smallest to largest). These zeros are the key numbers of the polynomial.

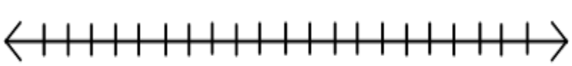
2. Use the key numbers of the polynomial to determine its test intervals.

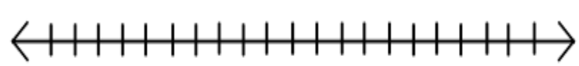
3. Choose one representative x-value in each test interval and evaluate the polynomial at that value. If the value of the polynomial is negative, the polynomial will have negative values for every x-value in the interval. If the value of the polynomial is positive, the polynomial will have positive values for every x-value in the interval.

1. Solve and graph each polynomial inequality.

a. $x^2 - x - 20 < 0$

b. $3x^3 - x^2 - 12x > -4$





2. Solve each of the polynomial inequalities.

a. $x^2 + 6x + 9 > 0$

b. $x^2 + 4x + 4 \leq 0$

c. $x^2 - 6x + 9 < 0$

d. $x^2 - 2x + 1 > 0$

RATIONAL INEQUALITIES

Rational expressions can change signs only at its zeros and its undefined values. These two types of numbers make up the key numbers of a rational inequality. When solving a rational inequality, begin by writing the inequality in general form with the rational expression on the left and the zero on the right.

3. Solve each rational inequality.

a. $\frac{x-2}{x-3} \leq -3$

b. $\frac{4x-1}{x-6} > 3$

APPLICATIONS

4. The marketing department of a calculator manufacturer has determined that the demand for a new model of calculator. The revenue and cost equations for the product are $R = x(60 - 0.0001x)$ and $C = 12x + 1{,}800{,}000$, where R and C are measured in dollars and x represents the number of calculators sold.

How many units much be sold to obtain a profit of at least $3,600,000?
NOTE: Profit = Revenue – Cost

5. Find the domain of $\sqrt{x^2 - 7x + 10}$.