To add or subtract rational expressions, you can use the least common denominator (LCD) method or the basic definition $\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}, b \neq 0, d \neq 0$. This definition provides an efficient way of adding or subtracting TWO fractions that have no common factors in their denominators.

6. Combine the rational expressions.

a. $\frac{x}{2x-1} - \frac{1}{x+2}$

$$\frac{X}{2X-1} \cdot \frac{X+2}{X+2} - \frac{1}{X+2} \cdot \frac{2X-1}{2X-1}$$

$$\frac{X(X+2)-(2X-1)}{(2X-1)(X+2)}$$

$$\frac{X^2+2X-2X+2}{(2X-1)(X+2)} = \boxed{\frac{X^2+2}{(2X-1)(X+2)}}$$

b. $\frac{4}{x} - \frac{x+5}{x^2-4} + \frac{4}{x+2}$

$$\frac{4}{X} - \frac{X+5}{(X+2)(X-2)} + \frac{4}{X+2}$$

$$\frac{4}{X} \cdot \frac{(X+2)(X-2)}{(X+2)(X-2)} - \frac{X+5}{(X+2)(X-2)} \cdot \frac{X}{X} + \frac{4}{X+2} \cdot \frac{X(X-2)}{X(X-2)}$$

$$\frac{4X^2-16-X^2-5X+4X^2-8X}{X(X+2)(X-2)}$$

$$\boxed{\frac{7X^2-13X-16}{X(X+2)(X-2)}}$$

c. $\frac{x+1}{x^2-2x-35} + \frac{x+6}{x^2+7x+10}$

$$\frac{X+2}{X+2} \cdot \frac{X+1}{(X-7)(X+5)} + \frac{X+6}{(X+5)(X+2)} \cdot \frac{X-7}{X-7}$$

$$\frac{(X+1)(X+2)+(X+6)(X-7)}{(X-7)(X+5)(X+2)}$$

$$\frac{X^2+3X+2+X^2-X-42}{(X-7)(X+5)(X+2)} = \frac{2X^2+2X-40}{(X-7)(X+5)(X+2)}$$

$$= \boxed{\frac{2(X-4)}{(X-7)(X+2)}}$$

d. $\frac{2x}{1-2x} + \frac{3x}{2x+1} - \frac{3}{4x^2-1}$

$$\frac{2X}{-(2X-1)} + \frac{3X}{2X+1} - \frac{3}{(2X-1)(2X+1)}$$

$$\frac{-2X}{2X-1} \cdot \frac{2X+1}{2X+1} + \frac{3X}{2X+1} \cdot \frac{2X-1}{2X-1} - \frac{3}{(2X-1)(2X+1)}$$

$$\frac{-4X^2-2X+6X^2-3X-3}{(2X-1)(2X+1)}$$

$$\frac{2X^2-5X-3}{(2X-1)(2X+1)} = \frac{(2X+1)(X-3)}{(2X-1)(2X+1)} = \boxed{\frac{X-3}{2X-1}}$$

e. $\frac{x-23}{x^2-x-20} - \frac{2}{5-x}$

$$\frac{X-23}{(X-5)(X+4)} - \frac{2}{-(X-5)} \cdot \frac{X+4}{X+4}$$

$$\frac{X-23}{(X-5)(X+4)} + \frac{2X+8}{(X-5)(X+4)}$$

$$\frac{3X-15}{(X-5)(X+4)}$$

$$\frac{3(X-5)}{(X-5)(X+4)} = \boxed{\frac{3}{X+4}}$$

f. $\frac{x-2}{(x+2)^2} + \frac{x}{5x+10} - \frac{1}{25}$

$$\frac{X-2}{(X+2)^2} + \frac{X}{5(X+2)} - \frac{1}{25}$$

$$\frac{X-2}{(X+2)^2} \cdot \frac{25}{25} + \frac{X}{5(X+2)} \cdot \frac{5(X+2)}{5(X+2)} - \frac{1}{25} \cdot \frac{(X+2)^2}{(X+2)^2}$$

$$\frac{25X-50+5X^2+10X-X^2-4X-4}{25(X+2)^2}$$

$$\boxed{\frac{4X^2+31X-54}{25(X+2)^2}}$$

COMPLEX FRACTIONS AND THE DIFFERENCE QUOTIENT

Fractional expressions with separate fractions in the numerator, denominator, or both are called **complex fractions**.

To simplify a complex fraction, combine the fractions in the numerator into a single fraction and then combine the fractions in the denominator into a one fraction. Then invert the denominator and multiply.

7. Simplify the complex fraction.

a. $\dfrac{\frac{1}{x+2}+1}{\frac{x}{3}-1}$

$$\frac{\frac{1}{x+2}+\frac{x+2}{x+2}}{\frac{x}{3}-\frac{3}{3}}$$

$$\frac{\frac{x+3}{x+2}}{\frac{x-3}{3}} = \frac{x+3}{x+2}\cdot\frac{3}{x-3} = \boxed{\frac{3(x+3)}{(x+2)(x-3)}}$$

b. $\dfrac{2+\frac{10}{x}}{1-\frac{25}{x^2}}$

$$\frac{\frac{2x}{x}+\frac{10}{x}}{\frac{x^2}{x^2}-\frac{25}{x^2}}$$

$$\frac{\frac{2x+10}{x}}{\frac{x^2-25}{x^2}} = \frac{2x+10}{x}\cdot\frac{x^2}{x^2-25} = \frac{2(x+5)}{x}\cdot\frac{x^2}{(x-5)(x+5)}$$

$$\boxed{\frac{2x}{x-5}}$$

To simplify an expression with negative exponents, factor out the common factor with the smaller exponent.

NOTE: When you factor, you subtract exponents.

8. Simplify the expression and make sure all the exponents are positive.

a. $(x-1)^{-\frac{1}{3}} - x(x-1)^{-\frac{4}{3}}$

$$\frac{1}{(x-1)^{1/3}} - \frac{x}{(x-1)^{4/3}}$$

$$\frac{1}{\sqrt[3]{x-1}} - \frac{x}{\sqrt[3]{(x-1)^4}}$$

$$\frac{1}{\sqrt[3]{x-1}} - \frac{x}{(x-1)\sqrt[3]{x-1}}$$

$$\frac{x-1-x}{(x-1)\sqrt[3]{x-1}}$$

$$\boxed{\frac{-1}{(x-1)\sqrt[3]{x-1}}}$$

b. $\dfrac{(4-x^2)^{\frac{1}{2}}+x^2(4-x^2)^{-\frac{1}{2}}}{4-x^2}$

$$\frac{(4-x^2)^{1/2}+x^2}{(4-x^2)(4-x^2)^{1/2}}$$

$$\frac{(4-x^2)^{1/2}+x^2}{(4-x^2)^{3/2}}$$

$$\frac{\sqrt{4-x^2}+x^2}{\left(\sqrt{4-x^2}\right)^3}$$

$$\boxed{\frac{\sqrt{4-x^2}+x^2}{(4-x^2)\sqrt{4-x^2}}}$$

9. Rewrite the expression by rationalizing the numerator.

a. $\dfrac{\sqrt{9+h}-3}{h}\cdot\dfrac{\sqrt{9+h}+3}{\sqrt{9+h}+3}$

$$\frac{9+h+3\sqrt{9+h}-3\sqrt{9+h}-9}{h(\sqrt{9+h}+3)}$$

$$\frac{h}{h(\sqrt{9+h}+3)} = \boxed{\frac{1}{\sqrt{9+h}+3}}$$

b. $\dfrac{\sqrt{x+2}-\sqrt{x}}{2}\cdot\dfrac{\sqrt{x+2}+\sqrt{x}}{\sqrt{x+2}+\sqrt{x}}$

$$\frac{x+2+\sqrt{x(x+2)}-\sqrt{x(x+2)}-x}{2(\sqrt{x+2}+\sqrt{x})}$$

$$\frac{2}{2(\sqrt{x+2}+\sqrt{x})} = \boxed{\frac{1}{\sqrt{x+2}+\sqrt{x}}}$$